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Alternative Approximations for Integrated Control/Structure Aeroservoelastic Synthesis

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A discussion of alternative complex pole and gust response approximations is presented, motivated by the need to extend nonlinear programming/approximation concepts optimization methodology from structural synthesis to the integrated control/structure synthesis of aeroservoelastic systems. Two new approximations are presented for the high-order, weakly damped linear time-invariant systems representing actively controlled airplanes in high-speed flight. They are compared with Taylor series, differential equations, and Rayleigh quotient approximations. The accuracy and computational efficiency of the alternative approximations are discussed. Although the development has been motivated by aeroservoelasticity and the examples used are from that domain, the new approximations are general and applicable to a wide range of control problems.

Introduction

STRONG interactions between active controls, structural dynamics, and aeroelasticity are common in modern aircraft[1] and must be taken into account as early as possible in the design process. The traditional approach to aeroservoelasticity has focused on avoiding undesirable interactions or eliminating them. In recent years the growing power of structural optimization,[2] aeroelastic tailoring,[3] and automated control synthesis techniques[4,5] has made it possible to harness these multidisciplinary interactions to benefit new designs.[6] The feasibility of aeroservoelastic optimization was discussed in Refs. 7–10. Furthermore, methods of integrated control/structural aeroservoelastic optimization, in which a realistic wing structure and its active control system are synthesized simultaneously to be optimal in some manner (while satisfying a wide range of structural, aeroelastic, and control system design constraints), are currently emerging.[9–13]

Combining nonlinear programming with approximation concepts (NLP/AC) has proven to be successful in solving structural optimization problems,[2,14] and it is important in aeroservoelastic optimization where the multidisciplinary interactions are complex and the analysis problems are computationally intensive. NLP formulation of the aeroservoelastic optimization problem is quite general. No a priori knowledge of the constraints that drive the design is needed. The approximation concepts are aimed at reducing the number of detailed analyses needed during optimization and at reducing its computational cost. In each stage of the NLP/AC design optimization process, a detailed analysis and the associated behavior sensitivity analysis are used for constructing approximations of the objective and constraint functions in terms of the design variables. The optimization subroutines use only approximate objective and constraint functions. Synthesis is carried out by solving a sequence of approximate optimization problems until convergence to some optimal solution is achieved. Each stage, consisting of a detailed analysis at a new base (reference) point in design space, behavior sensitivity analysis, approximate problem generation, and a solution of the approximate optimization problem, is counted as one "optimization cycle." The optimizer, of course, needs many function evaluations to converge in a stage, but these are based on the approximations and are computationally cheap.

To prevent the approximate problem from becoming too inaccurate as the design wanders away from the base point, additional constraints are added to the approximate optimization problem in the form of move limits on the design variables. For the NLP/AC approach to be practical, it is crucial to avoid too many detailed analyses for function evaluations and sensitivity calculations. This depends on making the approximations accurate yet simple enough for efficient solution.

Intuition and experience with structural systems led to the development of successful approximations for displacement, stress, and natural frequency constraints in terms of the structural design variables.[14] However, experience in aeroservoelastic system design[9–12] shows that, although move limits of 40 and 50% are allowed in optimization with structural design variables, only 5–20% move limits are needed when control system design variables are added, since the quality of eigenvalue and gust response approximations deteriorates rapidly as the design moves away from the base point used to construct these approximations. In some aeroservoelastic optimization problems, where the objective is to minimize some measure of gust response, move limits as low as 2–5% had to be used[11,12] to achieve convergence. Therefore effective new approximations, with improved accuracy, are needed for eigenvalue and gust response constraints, so that move limits can be increased. Larger move limits can be expected to improve overall efficiency of the optimization process.

Reference 15 contains a thorough discussion of approximations to eigenvalues of general matrices. The present paper addresses the problem of eigenvalue (closed-loop pole) and gust response approximations in the context of aeroservoelastic optimization. Only complex eigenvalue approximations that do not require eigenvector derivatives are considered here. The paper opens by describing the insights gained from aeroservoelastic optimization to date, and it identifies the probable causes of the poor performance of current approximations used for eigenvalue and gust response analyses. It then proceeds to suggest new approximations and examines them by studying typical exact and approximate eigenvalue and gust response constraints. Although the development has been motivated by aeroservoelasticity, and the examples used are from that domain, the methods presented are quite general and applicable to a wide range of control problems.

An important direction of research, described in Refs. 16 and 17, is aimed at developing accurate aeroservoelastic models of reduced order so that using detailed analysis throughout the optimization process will be practical. Still, approximations as described here can help accelerate the optimization of high-order aeroservoelastic and control augmented structural dynamic systems. When wing planform shape is varied in the

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design process, NLP/AC is even more important since the computational cost of the techniques of Refs. 16 and 17 becomes very high.

Complex Eigenvalue and Gust Response Analyses and Approximations Currently Used in Aeroservoelastic Optimization

To bridge the gap between aeroelastic and control system analysis techniques and pose the aeroservoelastic synthesis problem in a form suitable for modern control synthesis techniques, the root locus approach to flutter analysis[18] was adopted and extended to aeroservoelastic stability analysis. The closed-loop aeroservoelastic system, including its structural dynamic, unsteady aerodynamic, and control system elements, is modeled as a finite dimensional linear time invariant system (LTI) in state space. Its Laplace transformed linearized equations of motion are

$$s[B]\{x(s)\} = [A]\{x(s)\} + \{F\}w(s) \tag{1}$$

where $\{x(s)\}$ is the Laplace transformed state vector, $w(s)$ is a Gaussian zero mean white noise input,[19,20] and the real nonsymmetric matrices A and B depend on structural and control system design variables.

Aeroservoelastic stability analysis is carried out by solving the generalized eigenvalue problem

$$\lambda[B]\{\phi\} = [A]\{\phi\} \tag{2}$$

for the eigenvalues (closed-loop poles) λ_i, and the corresponding right eigenvectors $\{\phi_i\}$.

Damped natural frequency ω_i and damping measure ζ_i are determined from the real and imaginary parts of the complex eigenvalues:

$$\omega_i = \mathrm{Im}(\lambda_i) \tag{3}$$

$$\zeta_i = \frac{\sigma_i}{\sqrt{\sigma_i^2 + \omega_i^2}} \tag{4}$$

where

$$\sigma_i = \mathrm{Re}(\lambda_i) \tag{5}$$

Clearly, stability of the closed-loop system is lost when one or more damping measures become positive.

Gust response analysis in Refs. 9–12 is carried out (following Refs. 4, 19, and 20) by transforming Eq. (1) into standard LTI state space form

$$s\{x(s)\} = [\tilde{A}]\{x(s)\} + [\tilde{F}]w(s) \tag{6}$$

where

$$[\tilde{A}] = [B]^{-1}[A] \tag{7}$$

$$[\tilde{F}] = [B]^{-1}\{F\} \tag{8}$$

The state covariance matrix $[X]$ is then the solution of a matrix Lyapunov's equation[20]

$$[\tilde{A}][X] + [X][\tilde{A}]^T = -\{\tilde{F}\}[Q_w]\{\tilde{F}\}^T \tag{9}$$

where $[Q_w]$ is the intensity matrix of the white noise (of order 1×1 in this case). In the multidisciplinary synthesis capability of Refs. 9–12 and many other aeroservoelastic analysis codes, the QZ method[21] and the Hessenberg-Shur algorithm[22] are used to solve Eqs. (2) and (9), respectively.

Analytical behavior sensitivities are calculated in Refs. 9–12 based on the implicit differentiation of Eqs. (2) and (9) with respect to either structural or control system design variables. These analytical sensitivities are used to construct explicit first-order Taylor series approximations of eigenvalues and root mean square (rms) of gust response quantities in terms of direct or reciprocal design variables.[23] Exploratory parametric and optimization studies reported in Refs. 11 and 12 concluded that the quality of these eigenvalue and gust response approximations was sufficient to make aeroservoelastic optimization feasible. However, as already described in the introduction, their performance within the approximation concepts based optimization process was found to be quite disappointing, making it necessary to impose very strict move limits, delaying convergence, and leading to oscillatory behavior of the design with additional optimization cycles in some problems.

When first-order Rayleigh quotient approximations (first order RQA)[24–26] replaced the Taylor series based eigenvalue approximations, no improvement in convergence rate of the optimization process was noticed and larger move limits could not be allowed. However, it was interesting to find out that sometimes RQA eigenvalue approximations made convergence possible where Taylor series approximations failed and vice versa.

Figure 1 illustrates a major problem encountered when using reciprocal and direct approximations for gust response quantities. The figure shows a parametric study of the rms control surface rotation in atmospheric turbulence when a control system roll gain is varied in a YF16-type actively controlled wing. Both direct and reciprocal Taylor series approximations are quite good near the reference point where exact analysis and sensitivity information is available. When the roll loop gain is increased toward 3.5 times of its reference value, the aeroservoelastic system becomes unstable. This results in a sharp increase in aileron activity as the damping in the destabilized mode goes to zero. Neither the direct nor the reciprocal Taylor series approximation is capable of capturing this increase in activity.

New Eigenvalue Approximations

Accuracy and convergence problems in optimization with complex eigenvalue constraints similar to those described herein were encountered in the work reported in Refs. 25 and 26 in the context of control augmented structural synthesis. A control augmented structural system in Refs. 25 and 26 is represented for stability analysis by a second-order matrix equation of the form

$$\{s^2[\hat{M}] + s[\hat{C}] + [\hat{K}]\}\{x\} = \{0\} \tag{10}$$

where $\hat{M}$, $\hat{C}$, and $\hat{K}$ are control augmented mass, damping, and stiffness matrices, respectively.

Equation (10) is transformed into first-order form by the transformation

$$\{x_1\} = \{x\} \tag{11}$$

$$\{x_2\} = s\{x\} = s\{x_1\} \tag{12}$$

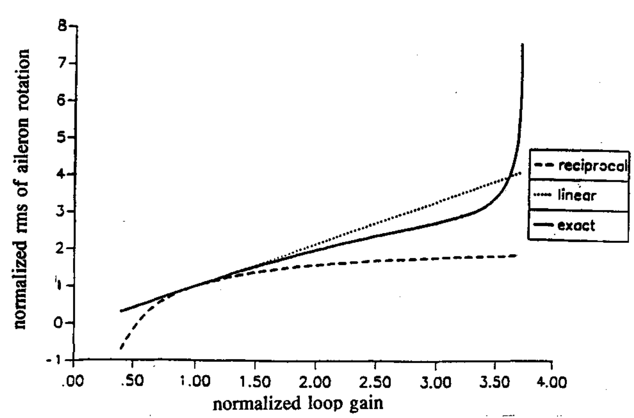

Fig. 1 Direct and reciprocal Taylor series approximations of the rms gust response of a lightweight fighter.

leading to a generalized eigenvalue problem of Eq. (2) where

$$[B] = \begin{bmatrix} \hat{M} & 0 \\ \hat{C} & \hat{M} \end{bmatrix} \tag{13}$$

$$[A] = \begin{bmatrix} 0 & \hat{M} \\ -\hat{K} & 0 \end{bmatrix} \tag{14}$$

and

$$\{\phi\} = \begin{Bmatrix} x_1 \\ \lambda x_1 \end{Bmatrix} \tag{15}$$

When the first-order Rayleigh quotient approximation is used, the eigenvalues are approximated by the expression

$$\lambda_i = \frac{\{\psi_i^0\}^T[A(p)]\{\phi_i^0\}}{\{\psi_i^0\}^T[B(p)]\{\phi_i^0\}} \tag{16}$$

where $\{\psi_i^0\}$ and $\{\phi_i^0\}$ are left and right eigenvectors of Eq. (2) fixed at their values at the reference (base) point in design space where detailed analysis was carried out, and the matrices $A(p)$ and $B(p)$ vary with changes in the vector of design variables p.

The observation of Ref. 26 is that whereas the first-order RQA assumes fixed left and right eigenvectors for moderate changes in the design, in actuality some elements of these eigenvectors depend directly on the eigenvalues [since the right eigenvectors contain both displacement and velocity components, Eqs. (12) and (15)]. Similarly, the left eigenvectors contain elements that depend directly on the eigenvalue

$$\{\psi\} = \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} \lambda y_2 \\ y_2 \end{Bmatrix} \tag{17}$$

If we assume, in an approximation based on the Rayleigh quotient, that only the x_1 and y_2 parts of the right and left eigenvectors are fixed at their reference values $\{x_1^0\}$ and $\{y_2^0\}$ whereas the eigenvalue-dependent parts vary with the eigenvalue; that is, if we use

$$\{\bar{\phi}\} = \begin{Bmatrix} x_1^0 \\ \lambda x_1^0 \end{Bmatrix} = \begin{Bmatrix} x_1^0 \\ 0 \end{Bmatrix} + \lambda \begin{Bmatrix} 0 \\ x_1^0 \end{Bmatrix} \tag{18}$$

$$\{\bar{\psi}\} = \begin{Bmatrix} \lambda y_2^0 \\ y_2^0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ y_2^0 \end{Bmatrix} + \lambda \begin{Bmatrix} y_2^0 \\ 0 \end{Bmatrix} \tag{19}$$

to generate a new approximation based on

$$\lambda_i = \frac{\{\bar{\psi}(\lambda_i)_i\}^T[A(p)]\{\bar{\phi}(\lambda_i)_i\}}{\{\bar{\psi}(\lambda_i)_i\}^T[B(p)]\{\bar{\phi}(\lambda_i)_i\}} \tag{20}$$

then a second-order equation for the approximate λ (Ref. 26) is obtained. This approximation, as shown in Ref. 26, performs much better than the first-order RQA in control augmented structural synthesis. A simple substitution of the whole fixed left eigenvector $\{\psi^0\}$ instead of $\{\bar{\psi}\}$ in Eq. (20) reveals that the same second-order equation for the approximate eigenvalue is obtained from Eq. (20) whether the whole left eigenvector is fixed at the reference value as in Eq. (16) or the eigenvalue-dependent left eigenvector [Eq. (19)] is used. Thus, Eq. (20) and the equation

$$\lambda_i = \frac{\{\psi_i^0\}^T[A(p)]\{\bar{\phi}(\lambda_i)_i\}}{\{\psi_i^0\}^T[B(p)]\{\bar{\phi}(\lambda_i)_i\}} \tag{21}$$

lead to the same result.

The aeroservoelastic formulation of Refs. 9 and 10 is more general than the formulation of Refs. 25 and 26. The control system is not modeled by just perfect displacement and damping actuators whose gains modify the stiffness and damping matrices, but rather by an array of control laws represented by transfer functions of any desired order, connecting sensor outputs to control surface actuators (Fig. 2). Sensor and actuator transfer functions are also included in the model.

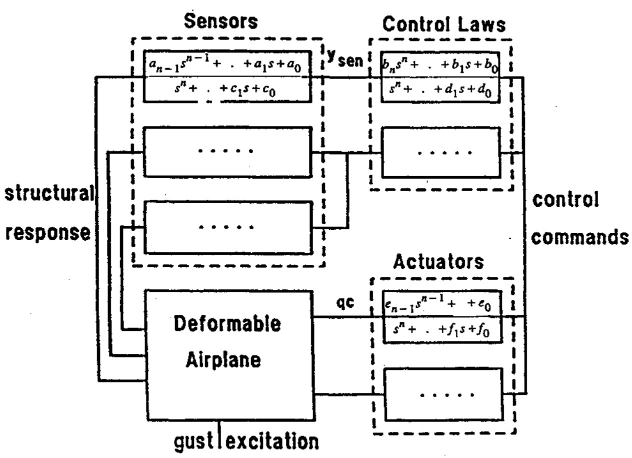

Fig. 2 Aeroservoelastic system.

A typical transfer function of a control element is given by

$$\frac{z(s)}{u(s)} = \frac{b_0 s^k + b_1 s^{k-1} + \cdots + b_k}{s^k + a_1 s^{k-1} + \cdots + a_k} \tag{22}$$

and translates to a state space model of the form

$$s\{x(s)\} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -a_k & -a_{k-1} & -a_{k-2} & \cdots & \cdots & -a_1 \end{bmatrix} \{x(s)\} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdots \\ 1 \end{bmatrix} U(s) \tag{23}$$

$$z = [b_k - a_k b_0 \mid b_{k-1} - a_{k-1} b_0 \mid \cdots b_1 - a_1 b_0]\{x\} + b_0 u \tag{24}$$

Interaction between control elements occurs when the output from one is the input to another. The input to each element affects only the last row of the matrix equation (23) of that element. It turns out that when control elements such as sensors, actuators, and control laws are coupled in the closed-loop system, still the relations between states associated with a certain element in the closed-loop system are the same as in the open-loop case, namely, for a control element of order n

$$x_2 = \lambda x_1 \tag{25}$$

$$x_3 = \lambda x_2 = \lambda^2 x_1 \tag{26}$$

$$x_4 = \lambda x_3 = \lambda^3 x_1 \tag{27}$$

and so on up to

$$x_n = \lambda^{n-1} x_1 \tag{28}$$

Thus, as in Eq. (18), there is a direct dependence of elements of the closed-loop right eigenvector [of Eq. (2)] on the corresponding eigenvalue up to the $n-1$th order, where n is the order of the highest order control element in the system. From among the structural response variables, the displacement part is fixed as in Eq. (18) and the velocity part is allowed to vary linearly with λ. Thus an approximation to the complex right eigenvector is obtained by

$$\{\bar{\phi}\} = \{\bar{\phi}^0\} + \lambda\{\bar{\phi}^1\} + \lambda^2\{\bar{\phi}^2\} + \cdots + \lambda^{n-1}\{\bar{\phi}^{n-1}\} \tag{29}$$

where $\{\bar{\phi}^i\}$ are complex vectors made of elements of the right eigenvector that are fixed at their values at the reference (base) design.

When the last state associated with each control element in the adjoint problem (and the second part of the structural state vector) is held fixed at its reference value, then it can be shown that elements of the left eigenvector of the closed-loop system depend on the eigenvalue λ, so that an approximate left eigenvector can be written as

$$\{\bar{\psi}\} = \{\bar{\psi}^0\} + \lambda\{\bar{\psi}^1\} + \lambda^2\{\bar{\psi}^2\} + \cdots + \lambda^{n-1}\{\bar{\psi}^{n-1}\} \tag{30}$$

where $\{\bar{\psi}^i\}$ are complex vectors depending on elements of the system matrices A and B and fixed elements of the left eigenvector.

Both Eqs. (20) and (21) [using Eqs. (29) and (30)] will lead now to the same new approximation for each λ_i that is based on a polynomial equation of order n

$$d_0\lambda^n + d_1\lambda^{n-1} + \cdots + d_n = 0 \tag{31}$$

and can be solved as an $n \times n$ generalized eigenvalue problem for the eigenvalue that is closest to the reference exact eigenvalue at the base design.

It is emphasized that, although the aeroservoelastic system may contain many states representing many structural modes, control elements of varying order and aerodynamic states, Eq. (31) is of nth order where n is the highest order of any transfer function in the open-loop system. Thus in a typical example the aeroservoelastic system equations, Eq. (2), may contain 30–100 states, whereas only eigenvalue problems of order 4 × 4 are solved for the approximation in Eq. (31), if the highest order control law is of order 4. The solution of these small problems is very fast compared with the solution of the full-order eigenvalue problem. Although somewhat more complicated than the direct and reciprocal approximations or the first-order RQA, this new Rayleigh quotient approximation (designated variable-order RQA) is aimed at capturing effects on the Rayleigh quotients caused by elements of the right and left eigenvectors that depend on higher powers of the eigenvalue.

The updated matrices $A(p)$ and $B(p)$ [Eqs. (16) and (20)] can be obtained by using first-order Taylor series in terms of direct or intermediate design variables[25–27]

$$[A] = [A_0] + \sum_i \left[\frac{\partial A}{\partial p_i}\right](p_i - p_i^0) \tag{32}$$

$$[B] = [B_0] + \sum_i \left[\frac{\partial B}{\partial p_i}\right](p_i - p_i^0) \tag{33}$$

The A and B matrices, however, may be nonlinear in some design variables. Examination of Eq. (24) reveals that if the numerator coefficient b_0 and any denominator coefficients a_i are included as design variables, the A matrix is nonlinear in the design variables because of the presence of products a_ib_0. In many cases it is not too expensive computationally to calculate updated A and B matrices exactly, especially when a set of fixed structural modes is used for structural order reduction.[9,16] The exact A and B matrices can then be used in the RQA and variable-order RQA expressions.

New Gust Response Analysis and Approximations

The main problem with Taylor series direct and reciprocal first-order approximations of gust response quantities, it should be remembered, is their difficulty in capturing the increase in gust response, when system design variables are varied and the real part of one or more eigenvalues (poles) tends to zero on the way to instability. A similar problem can be found with Taylor series approximations to the steady-state dynamic response of a system to sinusoidal excitation.[28] This suggests the use of left and right eigenvectors of the aeroservoelastic equation (2) to transform Eq. (1) to diagonal form so that the covariance matrix can be expressed explicitly in terms of the eigenvalues.

For a stable system the state covariance matrix is then

$$[X] = [\Phi][Y][\Phi]^T \tag{34}$$

where $[\Phi]$ is the complex matrix of column-by-column right eigenvectors. The elements of the matrix $[Y]$ are given by

$$Y_{i,j} = \frac{-(\{[\Psi]^T\{F\}\}[Q_w]\{[\Psi]^T\{F\}\}^T)_{i,j}}{\lambda_i + \lambda_j} \tag{35}$$

Proper pole placement (in terms of requirements on damping and frequency) is extremely important to guarantee stability and shape dynamic response, and it is still widely used in dynamic aeroelasticity. Besides, if the system is unstable, any gust response calculation is meaningless. Therefore stability analysis and behavior sensitivity analysis [Eq. (2)] in aeroservoelastic synthesis are almost always carried out before any gust response calculations. Thus the left and right eigenvectors and the eigenvalues of Eq. (2) are available before gust response analysis is carried out. Therefore, Eq. (35) can be used instead of Eq. (9) for the calculation of the covariance matrix at the reference (base) design point about which an approximate problem is to be constructed (provided the eigenvalue problem is well conditioned).

Based on Eqs. (34) and (35) two alternative covariance matrix approximations are suggested. The numerator terms of Eq. (35) are the same in both where the left eigenvector matrix $[\Psi]$ is replaced by the fixed matrix $[\Psi^0]$ obtained from the exact eigenvalue analysis at the base point in design space. The two approximations differ in the approximate eigenvalues that replace the exact eigenvalues in the denominator of Eq. (35) and in the approximate right eigenvector matrix that replaces the $[\Phi]$ matrix in Eq. (34). These two approximations are as follows.

1) The $[\Phi]$ is replaced by the fixed $[\Phi^0]$ [Eq. (16)], and the approximate eigenvalues used in Eq. (35) are obtained from first-order RQA [Eq. (16)]. This approximation is based on the same assumptions as in the first-order RQA for eigenvalues.

2) The $[\Phi]$ is replaced by the eigenvalue-dependent $[\bar{\Phi}(\lambda)]$ [Eqs. (18) and (25–29)], whereas the approximate eigenvalues used are obtained from the new RQA. This gust response approximation is consistent with the new Rayleigh quotient eigenvalue approximations.

The success of these two approximations in capturing the rise in gust response when a pole becomes destabilized depends on the quality of approximate eigenvalues in the denominator of Eq. (35).

There is no need to prepare approximations of all eigenvalues as might be implied by Eqs. (34) and (35). Based on a reference base design, approximations of eigenvalues can be generated for only the active and very weakly damped poles. These are used in Eq. (35) together with the fixed reference (base) values for the rest of the poles. Techniques of modal cost analysis[29] can also be used for further model order reduction or for identification of dominant poles whose variation should be approximated in Eq. (35). These are, however, beyond the scope of the this paper.

Computational Cost

When compared with simple direct or reciprocal first-order Taylor series approximations, the improved accuracy of higher order and more sophisticated approximations based on proper selection of intermediate response quantities[28] comes usually with an added cost due to longer computation times. In some cases the computation of a better approximation costs almost as much as a detailed new analysis.[15] Attention must be paid therefore to the computational cost of any new approximations.

Left and right eigenvectors are needed for calculating analytical sensitivities of the exact eigenvalues. Since these are needed for stability analysis, we assume that they are calculated during the detailed eigenvalue analysis in addition to the eigenvalues themselves. With these fixed right and left eigenvectors, the first-order RQAs are cheap to calculate. They require $2(N^2 + N) + 1$ complex operations per eigenvalue, not including the calculations of updated A and B matrices [Eqs. (32) and (33)], but assuming that the updated A and B matrices are fully populated. Here it is understood that N denotes the dimensions of the sparse matrices A and B associated with the first-order formulation [see Eqs. (2), (13) and (14)].

Computational cost of the variable-order RQA depends on the order of the highest order control system transfer function and the number and order of all control elements. The generation of coefficients for the nth order polynomial [Eq. (31)] involves triple products of the form [see Eq. (21)] $\{\psi_i^0\}^T[A]\{\bar{\phi}_i^k\}$ and $\{\psi_i^0\}^T[B]\{\bar{\phi}_i^k\}$ where the fixed left eigenvector $\{\psi_i^0\}$ is used along with the $\{\bar{\phi}_i^k\}$ sequence of vectors [Eq. (29), $k = 0, 1, \ldots, n-1$]. There are thus $2n$ triple products, each involving $N^2 + N$ complex operations. Roots of the nth order polynomial [Eq. (31)] are obtained from a $n \times n$ complex eigenvalue problem requiring about $10n^3$ complex operations. The evaluation of each updated eigenvalue using the new RQA requires about $2n(N^2 + N) + 10n^3$ complex operations. The new variable-order RQA will be cheap computationally in comparison with the exact eigenvalue solution, when n (the order of the highest order element in the control system) is small enough, and the number of active eigenvalue constraints are not large.

Equations (34) and (35) reveal that we need N^2 complex operations to generate the vector $[\Psi]^T\{F\}$ and then another $2N^2 + N$ complex operations to calculate the matrix Y (the matrix $[Q_w]$ is of order 1×1 in our case). The triple matrix product in Eq. (34) involves $2N^3$ complex operations. Thus the new gust response approximation, even when taking advantage of the already available eigenvalues and eigenvectors, seems to take almost as many operations as the detailed solution.[22]

Considerable savings in terms of computer time can be achieved if we realize that usually in aeroservoelastic synthesis only a few aileron activity and system response measures are considered. Each output quantity z_l corresponding to a gust response constraint can be expressed as

$$z_l = \{c_l\}^T\{x\} \tag{36}$$

so that the mean square response is given by

$$\sigma_{z_l}^2 = \{c_l\}^T[X]\{c_l\} = \{c_l\}^T[\Phi][Y][\Phi]^T\{c_l\} \tag{37}$$

When fixed right eigenvectors are used, the product $[\Phi_0]^T\{c_l\}$ can be prepared first for each output z_l (N^2 complex operations per z_l). Then, using the right-hand side of Eq. (37) requires an additional $N^2 + N$ complex operation for $\sigma_{z_l}^2$. Altogether there are $5N^2 + 2N$ complex operations per response quantity z_l. When the gust response approximation is based on approximate eigenvalues and right eigenvectors obtained by the new variable-order RQA (right eigenvector matrix $[\Phi]$ explicitly dependent on the eigenvalues), more operations are needed in an amount that depends on the order of the highest order control element n and the number of high-order control elements in the system. Compared with the computational cost of solving the Lyapunov equation directly (in the order of $19N^3$ real operations), this computational cost is low, provided the number of response quantities z_l is kept small.

Test Case

For the present studies a mathematical model of a small remotely piloted vehicle, similar to the one described in Refs. 30–32, is used. Its planform geometry is shown in Fig. 3. A small control surface located at about 80% span is used for active flutter suppression. The control surface chord is 20% of the local wing chord, and it is driven by an actuator whose transfer function is preassigned as follows:

$$\frac{q_{c1}}{\delta_1} = \frac{1.7744728 \cdot 10^7}{(s + 180)(s^2 + 251s + 314^2)} \tag{38}$$

where q_{c1} and δ_1 are the actual control surface rotation and actuator command, respectively.

The main wing box structure, extending from root to tip spanwise and to 80% chordwise, is the structure to be synthesized. A wing tip pod is added to the wing, and it is simulated by two 2.5 kg masses at the forward and aft points of the tip. The wing box construction includes all aluminum cover skins. The skin thickness distribution is represented by a nine-term polynomial in x and y, whose terms are formed from the polynomial product $(1, x, x^2)$ $(1, y, y^2)$. The coefficients of these terms serve as the nine structural design variables.

A Dryden gust model is used. Following Refs. 5 and 32, an accelerometer is placed on the wing strip containing the control surface. It is located in the middle (spanwise) and 0.65 chord point of the strip. Its measurement serves as an input to a control law that, in turn, generates an input command δ to the actuator of the wing control surface. Flutter, gust, and aeroservoelastic stability calculations are carried out at sea level and Mach 0.9 for the cantilevered wing in the examples described herein.

Two control laws are used for this study. The first is the localized damping type transfer function (LDTTF) described in Ref. 32. This low-order control law provides damping "locally" in the range of frequencies where damping is needed. Its form is

$$\frac{\delta}{y_{SE}} = \frac{b_2}{s^2 + a_1 s + a_2} \tag{39}$$

where y_{SE} is the accelerometer measurement and a_1, a_2, and b_2 are control system design variables. The second control law is a fourth-order law

$$\frac{\delta}{y_{SE}} = \frac{b_0 s^4 + b_1 s^3 + b_2 s^2 + b_3 s + b_4}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4} \tag{40}$$

The preassigned accelerometer transfer function is

$$\frac{y_{SE}}{\ddot{w}_{0.65c}} = \frac{314^2}{s^2 + 376.8s + 314^2} \tag{41}$$

where $\ddot{w}_{0.65c}$ is the actual acceleration at the measurement point.

For preliminary studies of approximation accuracy, focusing on control system design variables, the aeroservoelastic model used was first limited to one structural mode, a third-order actuator, a second-order control law, a second-order sensor, a second-order Dryden gust filter, one lag term in the

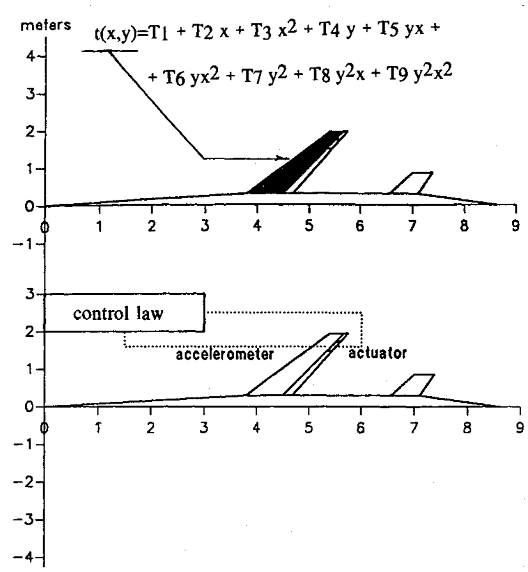

Fig. 3 Remotely piloted vehicle.

Roger approximation of unsteady aerodynamic loads, and one lag term in the Roger approximation of gust unsteady aerodynamic loads. The full-order aeroelastic system [Eq. (2)] in this case is, then, 13 × 13. Since the highest order of any control element is 3 (the actuator in this case), the eigenvalue problem to be solved for the new RQA is 3 × 3 [Eq. (31)]. More realistic aeroservoelastic systems are represented by seven structural modes and three lag terms in the Roger approximations, together with either a second order [Eq. (39)] or a fourth-order [Eq. (40)] control law yielding aeroservoelastic systems having dimensions 47 × 47 and 49 × 49, respectively. Accuracy of the new approximations in comparison with currently used approximations will be assessed by conducting parametric studies. In addition to the Taylor series and RQA approximations for complex poles, the recently introduced differential equations based approximations[27] are examined. Application of differential equations based approximations to structural natural frequencies (with symmetric A and B matrices and real eigenvalues) was introduced in Ref. 27. Generalization to the case of nonsymmetric A and B matrices and complex eigenvalues is straightforward. In all cases it is the real and imaginary parts of the eigenvalue itself that are approximated. The approximate damping measure is obtained

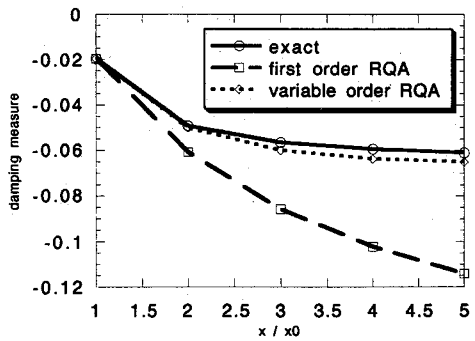

Fig. 4 RQA and new RQA damping measure variation of the 13 × 13 system.

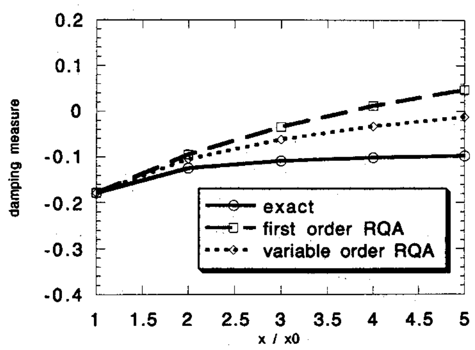

Fig. 5 RQA and new RQA damping measure variation of the 13 × 13 system.

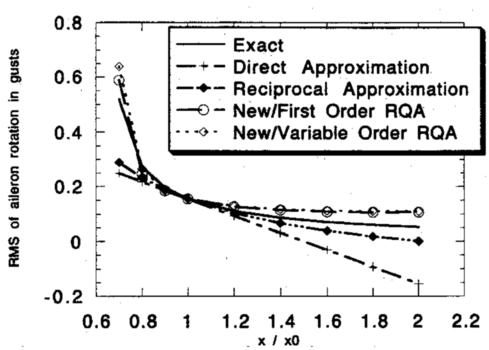

Fig. 6 Control surface gust response approximations of the 13 × 13 system.

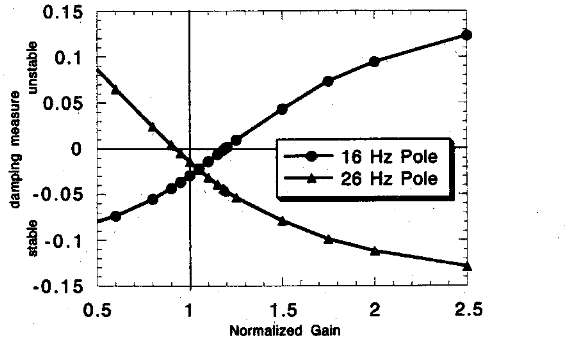

Fig. 7 Exact damping measure variation with a fourth-order control law (49 × 49 system, denominator coefficient of the s^3 term varied).

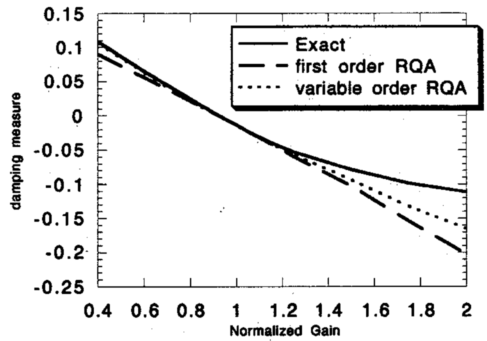

Fig. 8 26 Hz pole damping measure variation (fourth-order control law, 49 × 49 system, denominator coefficient of the s^3 term varied).

by using Eq. (4) with approximated real and imaginary parts of the eigenvalue.

Results

The variation (with changes in a typical control system design variable) of the damping measure ζ [Eq. (4)] associated with two weakly damped poles is examined in Figs. 4 and 5 for the 13 × 13 system with second-order control law. The design variable is the constant term a_2 in the denominator of Eq. (39), and it is varied from a reference value to five times that value. The variable-order RQA is compared with the first-order RQA and performs better in these two cases. For the case of Fig. 4, the quality of approximation obtained with the variable-order RQA is excellent over the whole range of design variable variation.

Comparison of conventional Taylor series direct and reciprocal first-order approximations for gust response with the new gust response approximations is shown in Fig. 6. The gust response quantity examined is the rms of aileron rotation q_c. Clearly the direct and reciprocal Taylor series approximations do not capture the rise in rms (q_c) as stability deteriorates. Both new gust response approximations seem to capture this rise very well on the conservative side.

Figure 7 shows the exact damping variation for two weakly damped poles in the 49 × 49 system. The fourth-order control law [Eq. (40)] is used in this case, and the parameter varied is the a_1 denominator coefficient. This is an interesting case since, as we move from the reference design in either positive or negative direction even with narrow move limits, the system becomes unstable due to two different instability mechanisms. The variable-order RQA is compared with the first-order RQA in Figs. 8 and 9 for the same two weakly damped poles. Accuracy of approximate damping measure is improved by the variable-order RQA, but the improvement is not dramatic. The new gust response approximations are examined in Fig. 10. There is good correlation between approximations and exact gust response when a_1 is reduced from its reference value. When a_1 is increased, both new gust approximations are

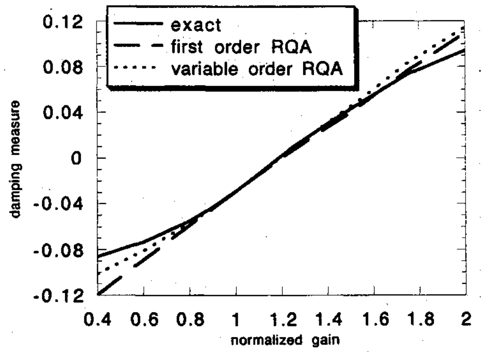

Fig. 9 16 Hz pole damping measure variation (fourth-order control law, 49 × 49 system, denominator coefficient of the s^3 term varied).

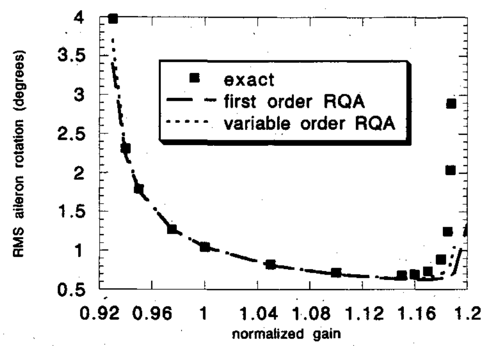

Fig. 10 Aileron gust response variation (fourth-order control law, 49 × 49 system, denominator coefficient of the s^3 term varied).

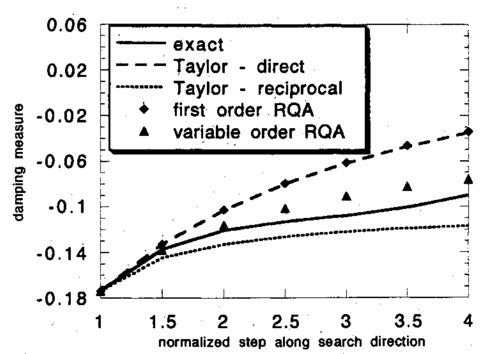

Fig. 11 26 Hz pole damping measure variation (second-order control law, 47 × 47 system, skin thickness and b_2 coefficient varied [Eq. (39)]).

unconservative. Clearly, however, the rise in aileron activity as the system approaches instability is captured.

We turn next to a case in which both structural and control system design variables are changed. Eigenvalue approximation accuracy for the 26 Hz pole of the 47 × 47 system (controlled by the second-order law) is examined in Figs. 11 and 12. The skin thickness and the b_2 coefficient of the second-order control law [Eq. (39)] are varied simultaneously by the same percentage change relative to some base design point values (this corresponds to a change in design along a given search direction). Taylor series direct and reciprocal approximations are compared with first-order and variable-order RQA for both frequency and damping. Although the reciprocal Taylor series approximation captures the damping trend quite well over a wide range of design variables, it performs poorly for the frequency. The variable-order RQA captures the frequency well and shows improvement over the direct Taylor series and first-order RQA in the case of damping measure.

Overall, in most of the many cases tested in this study, the new variable-order RQA performed as well as or better than the Taylor series, first-order RQA and differential equation approximations. In some cases, however, their performance was worse and no general rules were found that could identify these cases a priori. The new gust response approximations were always successful in capturing the rise in gust response associated with the loss of stability, and they were always superior to Taylor series approximation in that respect.

In all of the preceding cases, the system matrices A and B changed linearly with the selected design variables. We now turn to a case in which A and B are nonlinear in some design

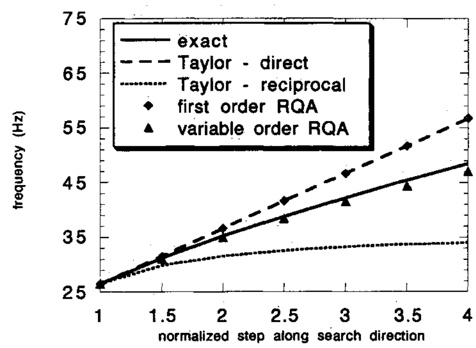

Fig. 12 26 Hz pole frequency variation (second-order control law, 47 × 47 system, skin thickness and b_2 coefficient varied [Eq. (39)]).

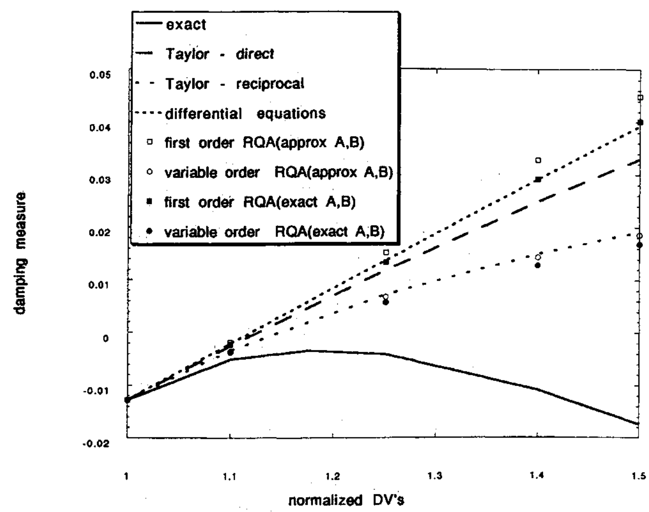

Fig. 13 26 Hz pole damping measure variation [fourth-order control law; 49 × 49 system; skin thickness, rear tip mass, and a_4 and b_0 coefficients varied in Eq. (40)].

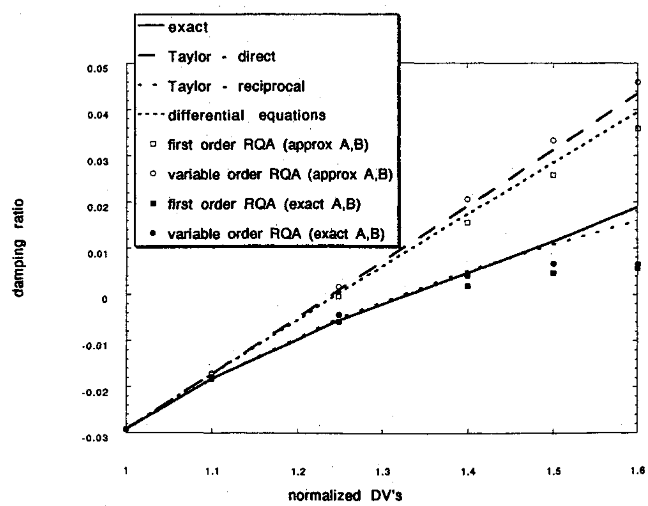

Fig. 14 16 Hz pole damping measure variation [fourth-order control law; 49 × 49 system; skin thickness, rear tip mass, and a_4 and b_0 coefficients varied in Eq. (40)].

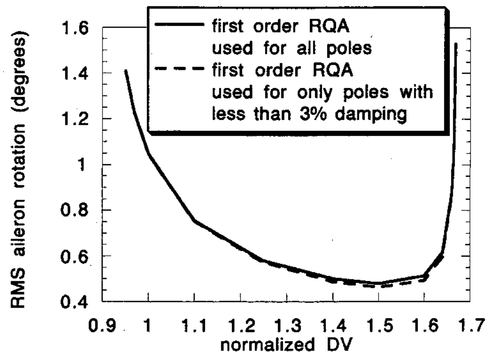

Fig. 15 Gust response approximations obtained by a) approximating the variation of all poles and b) freezing most poles at their reference (base) values and approximating the variation of only those that are very low damped (less than 3% damping).

variables. The performances of the first-order RQA and variable-order RQA are compared in the following two cases: 1) the updated A and B matrices are based on a linear approximation of the form of Eqs. (32) and (33) and 2) the exact A and B matrices are used corresponding to the various updated p_i. A nonlinear change of the A matrix is obtained here when both b_0 and any of the denominator coefficients of the fourth-order law [Eq. (40)] are changed simultaneously. Figures 13 and 14 show Taylor series direct and reciprocal approximations, differential equation based approximations, and first-order RQA and variable-order RQA approximations of the damping in two poles of the 49 × 49 system. Wing skin thickness, the tip trailing-edge concentrated mass, and the b_0, a_4 coefficients of the fourth-order control law are changed simultaneously.

Although using exact A and B instead of linear approximations of A and B in the first-order RQA and variable-order RQA does not lead to big differences in the case of the 26 Hz pole (Fig. 13), there is a major difference for the 16 Hz pole (Fig. 14). With the exact A and B matrices, both the first-order RQA and the variable-order RQA perform better than with linear approximations of A and B.

Again, no general rules were found to guide the user to one approximation or another, and it seems that performance of the new approximations is problem dependent.

Figure 15 shows new gust response approximations for the rms aileron rotation for the 49 × 49 system with variable b_0 term [Eq. (40)]. First-order RQAs are used in Eq. (35). In one case, all eigenvalues are approximated by their RQA values. In the second case, only two poles are approximated by the RQAs. These are the poles having less than 3% damping at the reference design. All other poles are held fixed at their reference (base) values. As the figure shows there is almost no difference between the cases. Toward instability, the new gust response approximation is dominated by the poles whose damping approaches zero. As long as these poles are allowed to vary in Eq. (35), the rise in rms gust response with loss of stability is captured.

Conclusion

Motivated by poor performance of currently used pole and gust response approximation, new pole and gust response approximations have been presented for aeroservoelastic synthesis. The new variable-order RQAs are aimed at improving eigenvalue approximations in aeroservoelastic LTI systems containing high-order control elements. The new gust response approximations take advantage of the availability of eigenvalues and eigenvectors calculated for aeroservoelastic stability analysis. By using an expression for rms gust response that explicitly depends on reciprocals of sums of eigenvalues, the rise in rms response as damping is being lost is captured by the new approximation. Computational aspects of the new approximations have been discussed, and their accuracy has been compared with other currently used approximations. In most test cases the variable-order RQA performed as well as or better than first-order Taylor series, first-order RQA, or differential equation based approximations. Their performance, however, was found to be problem dependent, and in some cases it was worse than other approximations. The new gust response approximation captures the increased gust response associated with reduced damping quite well. Since it needs eigenvalue approximations, its accuracy depends on the quality of approximate eigenvalues used. In some cases it is found to be unconservative, and the rise in gust response due to low damping is shifted with respect to the exact rise.

The development of the new approximations adds insight and contributes to understanding the problems associated with the application of nonlinear programming/approximation concepts to integrated aeroservoelastic synthesis. In many cases these new approximations offer a definite improvement over other approximations, and thus they should be added to the set of alternative approximations used for this complex multidisciplinary synthesis problem.

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